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NECESSARY AND SUFFICIENT CONDITIONS OF ABSOLUTE ASYMPTOTIC STABILITY OF LINEAR SYSTEMS OF DIFFERENTIAL EQUATIONS WITH CONSTANT DELAY

B. Koval' and Ye. Tsarkov

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16. Abstract

Necessary and sufficient conditions are determined of the asymptotic stability of a trivial solution for linear systems of differential equations and linear equations of the N-th order with arbitrary constant delay $\Delta > 0$.

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NECESSARY AND SUFFICIENT CONDITIONS OF ABSOLUTE ASYMPTOTIC STABILITY OF LINEAR SYSTEMS OF DIFFERENTIAL EQUATIONS WITH CONSTANT DELAY

B. Koval' and Ye. Tsarkov
Chernivets State University

Let us consider the differential equation /506*

$$\frac{dx}{dt} = Ax(t) + Bx(t-\Delta), \quad x\big|_{t<0} = \varphi(t), \tag{1}$$

where $A = ||a_{ij}||_1^N$, $B = ||b_{ij}||_1^N$ are true matrices with constant elements, $\Delta = \text{const} > 0$, $||\phi(t)||_N = \max\limits_{1 \leq i \leq N} |\phi_i(t)| < M (t < 0)$.

Let us assume that the trivial solution of equation (1) is absolutely asymptotically stable, because it is asymptotically stable at a rather constant delay of $\Delta > 0$.

In addition to (1), let us examine the theoretical system of differential equations

$$\frac{dy_k}{d\tau} = Ay_k + By_{k-1}, \; y_k \in R^N,$$
$$y_k(0) = y_{k0} \qquad (k = 0, \pm 1, \ldots), \tag{2}$$

where $y(0) = \{y_{k0}\}$ { m are dimensions of the finite sequences $\{y_k, k = 0 \pm 1, \ldots\}$ of n-dimensional vectors.

We may conclude that the trivial solution of equation (1) is absolutely asymptotically stable when, and only when, the trivial solution of system (2) is asymptotically stable (for the normal dimension of finite sequences) [1].

*Numbers in margin indicate pagination in original text.

Remarks. Let us consider (2) an equation in dimension m. If we switch to an equation conjugated with equation (2), we can easily see that to study the question of stability of the trivial solution, we can examine those equations having l_2, since $m \supset l_2 \supset l_1 = m^*$ [2]. /507

Theorem. For the trivial solution of equation (1) to be absolutely asymptotically stable, it is also necessary that the matrix

$$C = A + zB$$

is uniformly Hurwitzian at all z so that $|z| = 1$.

Let us multiply the kth unit of system of equations (2) by $z^k (|z| = 1)$ and, in compliance with the remarks, carry out a summation for k from $-\infty$ to ∞. Using this operation, we homomorphically transformed l_2 into a function space in an isolated neighborhood $|z| = 1$, summated with the square in terms of that contour; the transformation retains the norm with an accuracy to within the coefficient [3]. Hence, to explain the conditions of asymptotic stability of equation (2), we can test the equation

$$\frac{dY(z, \tau)}{d\tau} = (A + zB)\, Y(z, \tau), \tag{3}$$

where

$$Y(z, \tau) = \sum_{k=-\infty}^{\infty} y_k(\tau) z^k. \tag{4}$$

In fulfilling the conditions of the theorem, $\|Y(z, \tau)\|$ uniformly, with respect to $z(|z| = 1)$, approaches zero when $\tau \to \infty$. Taking the appearance of transform (4) into account, we can easily see that at very great τ simultaneously for all $||Y(z,0)|| < \delta$, the inequality $||Y(z,\tau)|| < \varepsilon$, if δ is selected properly.

Result 1.

For absolute asymptotic stability of the trivial solution to equation (1), matrix A must be Hurwitzian.

Result 2.

If matrix A is Hurwitzian diagonal and all subsequent major minors of the matrix

$$C_1' = \begin{pmatrix} 1 - \frac{|b_{11}|}{|a_{11}|} & -\frac{|b_{12}|}{|a_{22}|} \cdots & -\frac{|b_{1N}|}{|a_{NN}|} \\ -\frac{|b_{21}|}{|a_{11}|} & 1 - \frac{|b_{22}|}{|a_{22}|} \cdots & -\frac{|b_{2N}|}{|a_{NN}|} \\ \cdots & \cdots & \cdots \\ -\frac{|b_{N1}|}{|a_{11}|} & -\frac{|b_{N2}|}{|a_{22}|} \cdots & 1 - \frac{|b_{NN}|}{|a_{NN}|} \end{pmatrix}$$

are added, the trivial solution to equation (1) is absolutely asymptotically stable.

Actually, according to [4], the characteristic integers of the matrix $(\lambda E - A)^{-1}B$ do not predict the maximum intrinsic integer of the modulus matrix mod $(\lambda E - A)^{-1}B$. Using the properties of matrices with inherent elements, we can easily be convinced of the validity of Result 2.

Result 3.

Let us consider a differential equation of n^{th} order

$$x^{(N)}(t)+\sum_{i=0}^{N-1} a_i x^{(i)}(t)=\sum_{j=0}^{N-1} b_j x^{(j)}(t-\Delta). \tag{5}$$

We find that for absolute asymptotic stability of the trivial solution to equation (5), it is also necessary that /508

$$\min_{\omega} \frac{|P(i\omega)|}{|Q(i\omega)|}>1, \tag{6}$$

where $P(\lambda) = \lambda^N + a_{N-1}\lambda^{N-1} + \ldots + a_0$, $Q(\lambda) = b_{N-1}\lambda^{N-1} + b_{N-2}\lambda^{N-2} + \ldots + b_0$.

Requirement. According to the theorem, equation

$$P(\lambda)+zQ(\lambda)=0 \quad (|z|=1) \tag{7}$$

has no roots in the right half-plane. If at some $\omega = \omega_0$ there occurs an equation $P(i\omega) = Q(i\omega)$, then the root is imaginary in equation (7) when $z = -1$. Hence, the need arises for conditions (6).

Adequacy. Let relationship (6) and the integer ν be such that Re $\nu > 0$ and where $z = z_0$: $P(\nu) + z_0 Q(\nu) = 0$. Then $|P(\nu)| = |Q(\nu)|$; as a result of D-division properties [5], there exists a contradiction with assumption (6).

We will now show that the sufficient conditions of stability contained in study [6] come from equation (6). The fact that equation

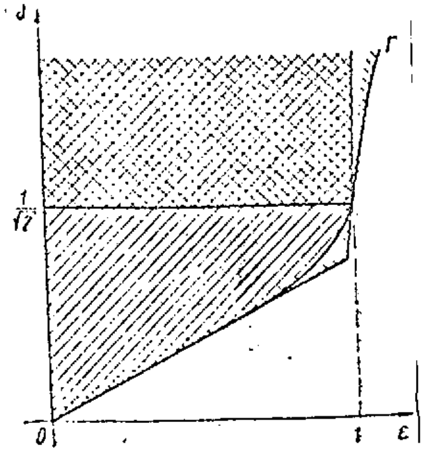

Fig. 4.

is Hurwitzian is derived from (7) where $z = 1$, while the conditions imposed on the coefficient of equation (6) of study [6] are derived from relationship (6).

As an example, let us find the region of absolute asymptotic stability of the equation which describes the motion of chassis axles as an airplane moves along a dirt airstrip [7]:

$$\ddot{\Theta}(t) + 2\delta\dot{\Theta}(t) + \Omega^2\Theta(t) + \xi\Omega^2\Theta(t-\Delta) = 0. \qquad (8)$$

According to (6)

$$\min_{\omega} \frac{|-\omega^2 + 2\delta i\omega + \Omega^2|}{\xi^2\Omega^4} > 1,$$

and the boundary range of stability is defined by the equation

$$\delta = \frac{1}{\sqrt{2}}\sqrt{1 - \sqrt{1-\xi^2}}.$$

In the figure ($\Omega^2 = 1$), the range of values of the parameter δ is shaded where the trivial solution to equation (8) is absolutely asymptotically stable; the criss-crossed shading depicts the range of stability derived with the aid of the theorem in study [6]. The author of study [7] incorrectly isolated the range of absolute asymptotic stability. Its boundary is depicted in the figure by meshed line G (Γ).

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